Spinoffs and Clustering

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Abstract

Geographic clustering of innovative industries is associated with the entry and success of spinoff firms. We develop a model to explain the multiple empirical patterns regarding cluster growth and spinoff formation and performance, without relying on agglomeration externalities. Clustering naturally follows from spinoffs locating near their parents. In our model, firms grow and spinoffs form through the discovery of new submarkets based on innovation. Rapid and successful innovation creates more opportunities for spinoff entry and drives a region's growth. Our model provides baseline estimates of levels of agglomeration that can be attributed to this process of innovation and spinoff formation.

KEYWORDS: Agglomeration, Clusters, Entry, Heritage, Innovation, Spinoffs

JEL classification codes: L25, O31, R12, R30

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1 Introduction

Geographic clustering of people and organizations is a fact of modern economic life. At the aggregate level, around half the world's population is located in cities. At the industry level, Ellison and Glaeser [1997] and Duranton and Overman [2005] show that in the modal manufacturing industry in the U.S. and U.K. respectively, plants are more clustered geographically than would be expected if they located randomly. These simple facts have widely been interpreted to reflect some sort of advantage of clustering. Wages and prices are higher in cities and in industry clusters such as Silicon Valley (Rosenthal and Strange [2004], Puga [2010]). Consequently, businesses in clusters must enjoy some kind of advantages in order to be competitive.

These advantages appear to extend well beyond the natural advantages some regions have for certain types of industries, such as the weather favoring the location of the movie industry in Hollywood (Ellison and Glaeser [1999]). New Economic Geography models (Fujita et al. [1999]) feature the role that costs of transporting goods play in inducing people and businesses to cluster in cities. Models of industry clustering commonly feature ideas proposed by Marshall [1890] about how clustering gives rise to agglomeration economies benefiting all firms located in clusters. These economies are related to the pooling of labor, the co-location of suppliers and producers, and localized spillovers of technological knowledge (Jaffe et al. [1993], Duranton and Puga [2004]). Although it is difficult to test the mechanisms underlying clustering, numerous studies find evidence consistent with the advantages of clusters, such as firms in clusters performing better, entry being concentrated in clusters, and firms in different industries clustering to a greater degree the more they share similar types of labor, inputs, and knowledge (Audretsch and Feldman [2004], Rosenthal and Strange [2004], Ellison et al. [2010], Greenstone et al. [2010], Puga [2010]).

In recent years new evidence has emerged about how industry clusters evolve that calls

out for explanation. Studies of the origins of firms in the automobile, tire, semiconductor, disk drive, and biotherapeutics industries, all of which were innovative in their time and evolved to be highly clustered, reveal a similar pattern. The regions where these industries ultimately clustered initially had one or at most a few related early successful firms. What distinguished them from other regions that also had successful early producers was that subsequently they grew through entry of new firms that were mainly spinoffs descended from their early successful producers. (We define spinoffs as entrants founded by individuals who previously worked for incumbent firms in the same industry. We refer to the firm where the primary founder last worked as the spinoff's "parent.") In a number of instances, the early successful firms receded but the region nonetheless prospered, propelled forward by successive generations of spinoffs. Geographic modellers are increasingly recognizing the importance of entrepreneurship for clustering (Glaeser et al. [2010]), but mainstream theories of clustering generally abstract from such forces and thus cannot readily address the accumulating evidence connecting clustering and spinoffs. Many studies of spinoffs in the last ten years have focused on innovative manufacturing industries, where clustering is also prominent (Feldman [1994], Audretsch and Feldman [2004]), and the growing body of empirical evidence regarding spinoff formation and performance is now attracting increased attention in its own right (see Klepper [2009b]).

The main purpose of this article is to organize the empirical evidence on spinoffs and clustering across industries and to develop a model to explain these stylized facts. Why do firms agglomerate especially in innovative industries? Why is so much of the entry driving the growth of agglomerative clusters coming in the form of spinoffs? And why are these spinoffs typically more successful, often becoming the industry leaders? We construct a simple theory of spinoffs that is related to firms growing over time through the discovery of new submarkets based on innovation. Spinoffs are assumed to locate close to their parents, as has been commonly found, in accordance with more general findings that new firms of

all kinds tend to locate close to where their founders have previously worked and resided (Figueiredo et al. [2002], Romanelli and Feldman [2006], Stam [2007]). We show that spinoffs (locating near their parents) naturally generate clustering.

Our model explains why the growth of an industry in a particularly concentrated region typically is marked by spinoffs, even if they only capture profits that would have gone to their parents. A simple insight powers the model. Innovation begets more innovation in a positive feedback cycle (Arthur [1990], Danneels [2002]), creating more new profit opportunities for more successful firms. When this dynamic gets going, rapid innovation opens the door for spinoffs to enter, driving the entire region's growth. Innovative capabilities are not directly observable, but when we see clusters of spinoffs, we get a strong signal about their innovative potential.

We posit that a firm's innovations build on the expertise it already has. Evolving innovative capabilities reflect organizational learning (Mitchell [2000]). Our model of new business opportunities growing out of existing ones is similar in spirit to recombinant growth (Weitzman [1998]), endogenous technological change (Klette and Kortum [2004]), combinatorial technological evolution (Arthur [2009]), and creative development (Feinstein [2015]). As in Klette and Kortum [2004], in particular, business units within a firm beget new business units. The new feature here is that newly created business units may be spun off into independent firms. As innovation makes spinoff entry possible, the implication is that spinoffs initially (upon entering the industry) produce products that are similar to their parents', and thus their performance correlates with their parents' performance. If an initial entrant in a region discovers a particularly rich vein of innovations, the spinoffs that descend there will be especially well-positioned for success.

Our model does not feature traditional agglomeration economies, but does include elements of such models as part of its structure. Indeed, in our model firms do not gain any advantages from being located in a booming cluster. We provide useful baseline estimates

of the Ellison-Glaeser index of agglomeration in the absence of such localized externalities. Empirical measures of agglomeration could be compared against this baseline to assess the impact of localized (pecuniary or non-pecuniary) externalities over and above what can be attributed to a natural process of innovation and spinoff formation.

The article is organized as follows. In Section 2 we relate the accumulating evidence about spinoffs and their role in industry clustering. Section 3 lays out our model. In Section 4 we show that the existence of spinoffs leads to clustering and offer an explanation why more innovative industries tend to be more highly clustered. In Section 5 we show how spinoff entry contributes to the growth of a cluster in a particular region. In Section 6 we show that our model is consistent with the empirical regularities about spinoff formation and performance. In Section 7 we offer baseline estimates of levels of agglomeration that can be attributed solely to the process of innovation and spinoff formation. Section 8 discusses the absence of agglomeration economies – and the prospect of incorporating them – in our model. The appendix contains proofs of all results as well as a glossary of symbols.

2 Industry Evidence

We consider the evolution of five U.S. industries that are well known for clustering: automobiles around Detroit, tires around Akron (Ohio), semiconductors and disk drives around Silicon Valley, and biotherapeutics around San Francisco, Boston, and San Diego. Each of these industries was highly innovative and grew greatly over time, attracting many entrants. Various studies have attempted to piece together the organizational and geographic heritage of the entrants in each of the industries to understand the forces giving rise to clustering. The picture that emerges is most comprehensive for autos, semiconductors, and disk drives, reflecting the availability of data on the periodic market shares of the leading producers. For the tire industry, only the heritage of producers in Ohio was traced, and for the biotherapeutics industry the analysis largely focused on the San Diego cluster. Despite

differences in coverage, the patterns in the five industries are remarkably similar. They are summarized in Table 1 as eleven stylized facts.

Patterns of Cluster Growth and Spinoff Entry

1. More innovative industries have more often become highly clustered.
2. Clusters typically were characterized by an early successful firm and then grew subsequently through entry.
3. A greater percentage of entrants in the clusters than elsewhere were spinoffs.
4. Spinoffs accounted for a disproportionate share of the leaders in the clusters relative to their share of entrants overall.
5. Clusters prospered after spinoffs entered, even while in some cases the flagship firm that seeded the region subsequently declined.
6. Spinoffs performed better than other entrants.
7. Larger firms spawned spinoffs at a higher rate.
8. Spinoffs from larger firms were superior performers.
9. Spinoffs that entered at a larger size tended to perform better.
10. Spinoffs in clusters outperformed spinoffs elsewhere.
11. Spinoffs initially produced similar types of products as their parents.

Table 1: Stylized facts based on studies of the automobile, tire, semiconductor, disk drive, and biotherapeutics industry clusters.

Fact #1 is inspired by the observation that the five industries chosen for their famous clusters have all experienced periods of rapid innovation. Indeed, this is no coincidence. Comparing across a host of industries, those that are more innovative – say, with greater total research and development expenditure as a percentage of sales – tend to be more concentrated geographically (Audretsch and Feldman [1996, 2004]).

Fact #2 indicates that the clusters in each of these five industries typically had a flagship early entrant and then grew over time through entry. For example, Olds Motor Works was the first great firm in the Detroit area. It entered in 1901 and soon became the largest firm in the industry, attaining a peak market share of 26% in 1905 . But Detroit was much more than Olds Motor Works. Over 100 firms entered in the Detroit area after Olds through 1924 (after which entry into the industry was negligible). By 1915, 11 of these firms were among

the largest 15 firms in the industry and the collective market share of the Detroit leaders was 83% (Klepper [2007, 2009a, 2010]). Fairchild Semiconductor was the analog of Olds Motor Works in the semiconductor industry. It entered in 1957 and grew to be the second largest firm in the industry by 1966, when its market share peaked at 13% . Over 100 other firms entered the semiconductor industry in the Silicon Valley area from 1957 to 1986, and as of 1985, eight of the top 16 firms with a collective market share of 42% were based in the Silicon Valley area (Klepper [2009a, 2010]). BF Goodrich and IBM were the flagship firms in the Akron and Silicon Valley clusters in the tire and disk drives industries respectively, both of which were spurred forward by entrants following the success of these two pioneers (Buenstorf and Klepper [2009], Christensen [1993], McKendrick et al. [2000]). Hybritech appears to have played a similar role in the San Diego biotherapeutics cluster (Mitton [1990]).

Facts #3 and 4 indicate that the entrants that fuelled the growth of the clusters were disproportionately spinoffs of indigenous producers.[1] The most extreme case was semiconductors. Every one of the leaders of the industry that were based in Silicon Valley was a spinoff of a Silicon Valley incumbent, which is not surprising given that nearly all the entrants in Silicon Valley were spinoffs of indigenous semiconductor firms. In contrast, most of the leaders of the industry based outside of Silicon Valley were experienced electronics producers and diversifiers from other industries, reflecting the much higher percentage of entrants outside Silicon Valley with these backgrounds (Klepper [2009a, 2010]). Similarly, in autos and disk drives a much higher percentage of the entrants in Detroit and Silicon Valley than elsewhere were spinoffs,[2] and nearly all the later leaders in the two clusters were spinoffs of indigenous firms (Klepper [2007, 2010], Agarwal et al. [2004], Franco

[1]There are always some entrants that are challenging to classify, and the various studies document how these cases were handled.

[2]Almost half of all spinoffs in the disk drive industry located in Silicon Valley (Kenney and von Burg [1999]), whereas early pioneers in the industry were distributed across many other cities, including Tulsa, Minneapolis, and San Antonio, and were especially prominent in Los Angeles (Christensen [1993]).

and Filson [2006]). The percentage of entrants that were spinoffs in the biotherapeutics industry was also markedly higher in the San Diego, San Francisco, and Boston clusters than elsewhere (Romanelli and Feldman [2006]), as was true as well for tire entrants originating around Akron versus the rest of Ohio (Buenstorf and Klepper [2009]). These firms predominantly originated from incumbent firms located close by.

Fact #5 reflects that clusters prospered (i.e., grew faster than other regions) after spinoffs entered and became industry leaders, even though in the autos, semiconductors, and disk drive clusters the early flagship companies in these regions declined (Klepper [2009a], Christensen [1993]). The semiconductor industry is particularly interesting because it allows for the construction of a counterfactual case involving an early flagship company in another region that did not give birth to so many spinoffs. The leading firm in the industry for many years was Texas Instruments (TI), which entered before Fairchild in 1952 in Dallas. TI and Fairchild were the pioneers of high performance silicon transistors and integrated circuits (ICs). Ultimately, Fairchild's management was overwhelmed by the wave of innovations it discovered, as discussed below. This tsunami sustained several spinoffs (and eventually sank the firm). TI had far fewer spinoffs than Fairchild (and did not decline like Fairchild) (Klepper [2009a]). Of course, Silicon Valley and not Dallas became the center of the industry, suggesting that the flood of spinoffs that emerged from Fairchild might actually have spurred the growth of the semiconductor cluster in Silicon Valley.

Fact #6 is that spinoffs were distinctive performers. In the automobile industry spinoffs accounted for about half of firms classified as leaders of the industry between 1895 and 1966 despite making up only 20% of the 725 entrants (Klepper [2007]). The performance of spinoffs is similar for semiconductor entrants (Klepper [2009a]). The disk drive industry is an extreme case, with nearly all of the leading producers as of 1989 having descended from IBM over a period of about 30 years (Franco and Filson [2006]). The available data for tires and biotherapeutics also seems consistent with this general pattern (Buenstorf and

Klepper [2009], Romanelli and Feldman [2006]).

Given the importance of spinoffs in all the industries, it is worthwhile to step back for a moment to discuss the forces contributing to spinoffs. The studies of these industries discuss the circumstances behind many of the leading spinoffs. Many seem to have been founded due to some kind of disagreement in the parent firm about new technological ideas or management practices. It was not uncommon for leading firms to be managed by technologists with limited ability to assess the market potential of new ideas (Agarwal et al. [2004], Lécuyer [2006], Klepper [2007, 2009a]). In other instances, individuals with limited industry experience gained control of leading firms; this occurred especially after the firms were acquired by firms in other industries (Klepper [2007, 2009a]). These circumstances conspired at times to make firms unwilling to pursue ideas that turned out to have significant market potential. Sometimes others outside the firm could better evaluate the prospects of these ideas and would sponsor efforts by employees that worked on the ideas to form their own spinoff companies to pursue the ideas (Klepper [2007, 2009a]). Fairchild illustrates these themes. It suffered three notable managerial mistakes. First, while managed by technologists with little management experience, it developed the first ICs but did not recognize their market potential (Moore and Davis [2004]). Second, it was controlled by a defense contractor with little appreciation for the semiconductor industry's use of stock options as incentives. And third, it established R&D and manufacturing at separate locations and had difficulty mediating conflict between the divisions, making innovating difficult (Bassett [2002]). Each of these disagreements caused top employees to leave and form spinoffs, including, famously, Intel (Lécuyer [2006]). Fairchild was perhaps an extreme case, but other firms such as Olds Motor Works suffered similar conflicts that led to many spinoffs (Klepper [2007]). Conflicts, even at well-established firms, are inevitable, and they appear to arise unpredictably.

Despite the ubiquity of disagreements in the origins of spinoffs, the existence of spinoffs

does not indicate incompetence at the parent firm. Quite the contrary. As facts #7 and 8 indicate, the largest firms spawned spinoffs at the highest rate, and their spinoffs were superior performers. Fairchild was the exemplar. Among the seven leading semiconductor producers in Silicon Valley other than Fairchild as of 1985, five were spinoffs of Fairchild and the other two were founded by employees of other semiconductor firms that previously had worked at Fairchild (Klepper [2009a, 2010]). Fairchild spawned so many spinoffs that its offspring were cleverly dubbed the Fairchildren.

IBM and its successful descendants had a similar effect on the disk drive industry.[3] Of 40 spinoffs that entered the disk drive industry by 1993, 28 had parents that ranked amongst the top ten leaders by market share at some point between 1976 and 1992, including nine out of the ten spinoffs that themselves made it onto this elite list (Franco and Filson [2006]). These firms also survived longer than spinoffs with less distinguished parents.[4] As an example of this star-studded lineage, the very top firm in 1992, Conner, was a spinoff of Seagate (the top firm for much of the late '80s), whose lineage traces through Shugart Associates, Memorex and eventually back to IBM, all industry leaders at some point (Franco and Filson [2006]).

In autos, all the leading spinoffs in the Detroit area either descended from Olds Motor Works or from three other early leaders, Cadillac, Ford, and Buick, that benefited from subcontracting from Olds (Klepper [2007]). Similarly, tire spinoffs founded by individuals that had worked for the big three Akron firms – Goodrich, Goodyear, and Firestone – survived markedly longer than other spinoffs (Buenstorf and Klepper [2009]). No information is reported about the performance of spinoffs in biotherapeutics, but similar to the other industries the lead early producer in the San Diego cluster, Hybritech, was a fertile source of spinoffs with 13 descendants over its first 10 years (Mitton [1990]).

[3]The spinoffs of a firm and all the spinoffs of its spinoffs are called the firm's descendants.

[4]Our calculations draw on data presented in Christensen's [1993] Table 3 and Figure 7, as well as Franco and Filson's [2006] Table 1.

A few other stylized facts based on more limited data can also be established. Fact #9 is based on data on the entry sizes of spinoffs that were compiled for the automobile and tire industries (Klepper [2007, 2010], Buenstorf and Klepper [2009]). In both industries, spinoffs that entered at larger sizes and had larger parents (at their time of entry) turned out to be superior performers. Fact #10, that spinoffs located in clusters performed better than spinoffs outside of these clusters, also relies on data on survival rates in the automobile and tire industries (Klepper [2007, 2010], Buenstorf and Klepper [2009]), and accords with a study of Dutch publishing firms located inside and outside of Amsterdam (Heebels and Boschma [2011]). Fact #11 is based on data that were compiled for the semiconductor and disk drive industries about the types of products that firms produced (Klepper et al. [2011], Franco and Filson [2006]).[5] Not surprisingly, spinoffs initially produced products more like those produced by their parents than other firms in their industry.

These eleven facts reflect the key patterns about spinoffs and clustering that our model will address.

3 Innovation and Industrial Evolution

An industry is assumed to be composed of niches or submarkets that are discovered through innovation. Each submarket possesses certain characteristics or attributes, and we can identify a submarket by specifying its attributes. There is an uncountably infinite set S of possible attributes, and we represent a submarket as a finite subset $x \subset S$. That is, the set of attributes present in a submarket $\{s : s \in x\}$ fully characterizes that submarket, x. We let $|x|$ denote the number of attributes that describe and together define submarket x.[6] For simplicity, we assume that the existence of one submarket has no effect on the demand or costs in other submarkets.

[5] Data on the laser industry also supports the overlap between the initial products of spinoffs and those of their parents (Klepper and Sleeper [2005]).

[6] We might think of this as the complexity of the submarket.

New submarkets are discovered through innovation. A firm may innovate on any of its submarkets by incorporating a single new attribute. So, a firm with expertise in submarket x may discover a new submarket $x' = x \cup \{s\}$ for any $s \in S \backslash x$. This does not destroy the pre-existing submarket x. The firm simply may expand into the new submarket x' as well. We assume that a continuous probability distribution on S determines which attribute $\underline{s}$ is incorporated in the discovery of a new submarket.[7]

Our core idea is that the process of innovation is based on firms building off of what they know. Because newly discovered attributes are combined with existing submarkets, a firm's innovative capabilities evolve as the firm gains experience in more submarkets. Firms thus expand into new submarkets that are related to submarkets with which they already have experience – related in the sense that they share one or more common attributes. This conforms with the insight that diversified firms generally develop products in related submarkets (Nelson and Winter [1982], Montgomery [1994]). Let $X_{j,t}$ denote the set of submarkets that firm j has entered at time t and $N_{j,t}$ denote the number of these submarkets. We will explain later how a firm enters the industry and discovers its first submarket.

Innovations (by incumbents) are discovered according to a continuous-time Poisson branching process with mean intensity λ. The parameter λ captures the rate of incumbent innovation in the industry. For each submarket in which a firm already produces, the probability of the firm discovering a new submarket in the time interval dt is λdt. Then the expected number of new submarkets firm j discovers in time interval dt is $N_{j,t}\lambda dt$. Thus, more diversified firms tend to discover innovations more rapidly.

The demand in a new submarket depends on the attributes of that submarket as well as some degree of randomness. We represent the inverse demand function in submarket x as

[7]Formally, we assume a non-atomic probability measure μ on S. That is, for any measurable set $\hat{S}$, the probability that the newly discovered attribute comes from this set is $\text{Prob}(\underline{s} \in \hat{S}) = \mu(\hat{S})$. The assumption that μ is non-atomic, along with the earlier assumption that S is uncountable, simply helps us avoid the case that the same exact submarket is discovered multiple times.

$p = \bar{p}(x) - mq$, where p is the price of the product, $q \geq 0$ is the total quantity demanded of the product (at any time), m is a parameter that sets the units for the quantity produced, and for any submarket x, the value of $\bar{p}(x)$ is a random draw on the price at which demand emerges. As described presently, the distribution of $\bar{p}(x)$ varies across submarkets (i.e., depends on the attributes they possess), but the random draws are independent (given these attributes). The unit cost of production in each submarket is k. A firm monopolizes any submarket it discovers and so produces the monopoly output $q_m(x) = \frac{\bar{p}(x)-k}{2m}$ (as long as this is non-negative) and charges the monopoly price $p_m(x) = \frac{\bar{p}(x)+k}{2}$, which yields a revenue stream of $\frac{(\bar{p}(x))^2-k^2}{4m}$ and a profit stream of $\frac{(\bar{p}(x)-k)^2}{4m}$.[8]

Let $\underline{\eta}(x) \equiv \max\left(\frac{\bar{p}(x)-k}{2m}, 0\right)$, so the firm's submarket output is $\underline{\eta}(x)$ and its profits are $m(\underline{\eta}(x))^2$. If it turns out that $\eta(x) = 0$, then the firm simply decides not to enter submarket x after all, and in the case of an arriving startup attempting to enter the industry, the firm would not be able to form. A firm's total output (or revenue / profit respectively) is simply the sum of the output (revenue / profit) it produces in each of its submarkets. So, letting $\pi_{j,t}$ denote the profit firm j earns at time t, we have

$$\pi_{j,t} = \sum_{x \in X_{j,t}} m\left(\underline{\eta}(x)\right)^2.$$

The value of $\underline{\eta}(x)$ is determined by the value of $\bar{p}(x)$. We find it convenient to characterize the distribution of $\underline{\eta}(x)$ directly rather than through that of $\bar{p}(x)$. To do so, we introduce parameters z_s indicating the quality of any attribute s. In general, we assume that the distribution of $\underline{\eta}(x)$ is strictly increasing (in the sense of shifting its cumulative distribution function strictly downward at every point) in z_s for each $s \in x$, i.e., demand (and profit) is increasing with the quality of each attribute of the submarket. We let S_z denote

[8] We could work out a similar profit function if the innovating firm were, say, a Stackelberg leader, but the monopolistic framework is simplest.

the set of attributes having quality z. We assume some heterogeneity in quality, i.e., that $\text{Prob}(\underline{s} \in S_z) < 1$ for all z. The quality parameters are not directly observable, but realized profits in a submarket provide a signal of their values. We also do assume a nontrivial probability that any submarket x fails (i.e., generates no demand), $0 < \text{Prob}\big(\underline{\eta}(x) = 0\big) < 1$.

A specific example of a distribution that satisfies our assumptions for $\underline{\eta}(x)$ is the mixed geometric distribution,

$$\text{Prob}\left(\underline{\eta}(x) = \eta\right) = \frac{1}{|x|} \sum_{s \in x} z_s^{\eta}(1 - z_s) \text{ for } \eta \in \{0, 1, 2, \ldots\}.$$

With this functional form assumption, the probability that submarket x is a success (i.e., generates positive demand) is simply the average of the quality parameters of its attributes, $\text{Prob}\big(\underline{\eta}(x) > 0\big) = \frac{1}{|x|} \sum_{s \in x} z_s = \bar{z}_x$. For this distribution we find it reasonable to restrict $z_s \in (0, \frac{1}{2}]$ for all s as an acknowledgement that most innovations fail. We will make clear when we rely on this specific functional form.

The realizations of output $\eta(x)$ in each submarket x are of course independent events. Yet, interestingly, related submarkets will appear to have correlated outputs (and profits) because they both depend on some of the same attributes. If $\underline{x}$ and $\underline{x'}$ refer to randomly chosen submarkets, then there will be positive correlation in their outputs $\underline{\eta}(\underline{x})$ and $\underline{\eta}(\underline{x'})$ whenever there is a possibility that these submarkets $\underline{x}$ and $\underline{x'}$ share common attributes.[9] Indeed, we have assumed that firms grow by discovering submarkets that share common attributes, so the profits in different submarkets discovered by the same firm will be positively correlated. The growth of a firm is path dependent because expansion into new submarkets depends on which submarkets it has already entered and because the profitability of these new submarkets correlates with the profitability of existing, related submarkets.

As discussed in Section 2, firms cannot always recognize the prospects of their innova-

[9]Moreover, the more closely related the submarkets (holding fixed their complexity), as captured by the fraction of shared attributes, the greater the correlation (in their values of $\underline{\eta}$ and) in their profits.

tions and sometimes fail to pursue promising submarkets. There are a myriad of reasons why this might occur, but our model does not require specification of the particular reason why an incumbent firm may be unable to enter a newly discovered submarket. We simply recognize that this may occur, and when it does, employees who contributed to the discovery of the innovation may then leave their old firm and form a spinoff (typically with financial support from another individual or company that can better evaluate the prospects of the innovation). Accordingly, we assume there is a probability $\alpha > 0$ that a firm will not pursue a newly discovered submarket x' (regardless of its attributes) and that employees at that firm break off on their own to pursue it. Then, if it turns out that $\eta(x') > 0$, they form a spinoff, which enters the submarket and hence the industry. The spinoff can continue to innovate on x', while the incumbent remains in the pre-existing submarket x. Denote the number of spinoffs firm j spawns during the interval (t, t') as $\sigma_j(t, t')$.[10]

Of course, new firms in the industry are sometimes formed by entrepreneurs with backgrounds outside of the industry. (Industry pioneers, for example, must by definition be outside entrants.) These outside startups, without expertise in any existing submarket, are assumed to enter an "entry-level" or single-attribute submarket with the attribute drawn from the aforementioned probability distribution on S. We let $\kappa(t)$ denote the mean arrival rate of outside startups at time t, with actual arrivals independent random events.

When a new firm enters the industry, it locates in one of R regions. We assume that the region in which an outside startup enters is random, with the probability of locating in any particular region r being equal to the share of economic activity in the region (across all industries), denoted as f_r. Spinoffs, however, locate in the same region as their parents. We will make use of the indicator variable $u_{j,r} = 1$ if firm j is located in region r and 0

[10]The mean rate at which firm j spawns spinoffs at time t is

$$\dot{\sigma}_{j,t} \equiv \lim_{\Delta t \to 0} E\left(\sigma_j(t, t+\Delta t) \,|\, X_{j,t}\right) = \sum_{x \in X_{j,t}} \lambda\alpha \int_S \text{Prob}\big(\underline{\eta}\,(x \cup \{s\}) > 0\big)\ \mu(ds). \tag{1}$$

otherwise. Aggregating by region simply involves summing over the firms in existence at time t, which we denote $\{1, \ldots, J_t\}$. So, $\varpi_{r,t} = \sum_j u_{j,r}\pi_{j,t}$ denotes the total profits in region r at time t, and $\varsigma_r(t,t') = \sum_j u_{j,r}\sigma_j(t,t')$ denotes the number of spinoffs forming in region r during the interval (t,t').[11] We denote the first entrant in region r as $\hat{\jmath}_r$.

The fundamental insight underlying the process of spinoff formation is that spinoffs originate within incumbent firms from new ideas. Because spinoff formation is tied to innovation and innovations incorporate some already-known attributes into new submarkets, our model has built in the feature that spinoffs initially produce products that are similar to their parents', i.e., that are in related submarkets (Fact #11). Of course, after a spinoff enters the industry, it can then continue to innovate on its own. While the spinoff's and parent's product lines will always share a common thread, they may gradually diverge.

We have accommodated Fact #11 directly, but the remaining facts in Table 1 require explanation. In the following sections we work through the implications of the model and relate them to these stylized facts. We first tackle the occurrence of clusters, a notable phenomenon in and of itself.

4 Clustering

In this section we examine the phenomenon of clustering. We show that the formation of spinoffs implies that we can expect some clustering, and we then address Fact #1.

Ellison and Glaeser [1997] point out that some degree of geographic concentration in an industry is to be expected as a result of the lumpiness of plants, apart from any agglomerative forces that might contribute to clustering. Intuitively, if there are only a limited number of plants, then an industry's activity must be unevenly distributed in geographic space. For simplicity, we assume that all firms operate only one plant. Let $\theta_{j,t}$ denote the

[11] The splitting probability α has no effect on regional profits (or output), but it does affect the number of firms in each region.

market share[12] of firm j at time t and let $\vartheta_{r,t} = \sum_j u_{j,r}\theta_{j,t}$ denote the market share of all firms located in region r at time t. Let $H_t = \sum_j \theta_{j,t}^2$ denote the industry Herfindahl index at time t. A standard measure of clustering at any point in time t is $L_t = \sum_r (\vartheta_{r,t} - f_r)^2$. Following Ellison and Glaeser, we can calculate the expected value of L_t, conditional on the firm-size distribution, in the case that (contrary to our assumption that spinoffs locate in the same region as their parent) all firms choose their locations independently with probability f_r of selecting region r.

Lemma 1 (Ellison and Glaeser (1997)) *If every firm were to independently locate in a region r according to the probability f_r, then*

$$E(L_t \mid J_t,\, \theta_{1,t}, \ldots, \theta_{J_t,t}) = \Big(1 - \sum_r f_r^2\Big) H_t.$$

Ellison and Glaeser thus propose an index of geographic concentration

$$\gamma_t = \frac{L_t - (1 - \sum_r f_r^2)\, H_t}{(1 - \sum_r f_r^2)\,(1 - H_t)}$$

that controls for the size distribution of firms. The index is normalized to have mean value $E(\gamma_t) = 0$ in the case that all firms choose their locations randomly (as if by throwing darts at a map) with no natural geographic advantages or industry-specific agglomerative forces. By Lemma 1, if there were no spinoffs, then there would be no clustering (beyond random fluctuation) in our model (i.e., we would have $E(\gamma_t) = 0$).

Using Ellison and Glaeser's index, we can establish that in our model, in which the only agglomerative force is the locational inertia of spinoffs, industries that have experienced spinoffs are expected to be clustered (see Corollary 1 below). Indeed, if we know the

[12] We could specify market share by output, revenue, or profit without affecting our results. We make use of the normalization $\sum_j \theta_{j,t} = 1$.

heritage of all firms in the industry, we can work out the expected value of the Ellison-Glaeser index. For any outside startup $j^{\mathtt{o}}$, define $\beta(j^{\mathtt{o}})$ to be the set including the firm and all of its descendants (i.e., its spinoffs and spinoffs of its spinoffs, etc.). Let $\hat{\theta}_{j^{\mathtt{o}},t} = \sum_{j \in \beta(j^{\mathtt{o}})} \theta_{j,t}$ denote the combined market share of these firms, and let $\hat{H}_t = \sum_{j^{\mathtt{o}}} \hat{\theta}^2_{j^{\mathtt{o}},t}$ (summing just over the outside startups).

Theorem 1

$$E\big(\gamma_t \;\big|\; J_t,\, \theta_{1,t}, \ldots, \theta_{J_t,t},\, \beta(j^{\mathtt{o}})_{\{1,\ldots,J_t\}}\big) = \frac{\hat{H}_t - H_t}{1 - H_t}.$$

As a corollary of Theorem 1, we have:

Corollary 1

$$E\Big(\gamma_t \;\Big|\; \sum_r \varsigma_r(0,t)\Big) > 0 \;\; \textit{if and only if} \;\; \sum_r \varsigma_r(0,t) > 0.$$

Corollary 1 states that conditional on the existence of (one or more) spinoffs, an industry has a positive expected level of geographic concentration. Intuitively, if spinoffs locate in the same region as their parent, then firms will be more clustered in regions with successful initial entrants than would be expected randomly. We can think of it as a special case of firms being attracted to regions by the presence of others – specifically it is spinoffs being "attracted" to regions by the presence of their parents.

Spinoff formation requires two events: discovery of a successful submarket and splitting the resulting activity off from the parent firm. Only the first event – the innovation – contributes to the growth of industrial activity in those regions that initially get ahead. Yet Ellison and Glaeser's perspective is to take the firm size distribution as given and to correct for it when measuring the industry's concentration. From that perspective, in effect controlling for submarket discovery, the latter event – splitting activity off from the parent

to form the spinoff – generates higher measured concentration only by distorting the correction for the finite sample of discrete-size firms. In our model, the profits generated by spinoffs could just as well have been generated by their parents if they had been able to follow through on their innovations, but the spinoff entry leads to the industry having more smaller firms and thus appearing to be more clustered because we overcount the number of independent location choices. Still, as we explain in the next section, spinoff entry indicates fertile ground for innovation, which tends to lead to continued growth.

Now that we have established that clustering is a feature of our model, we can address the pattern that more innovative industries tend to become more clustered.

Theorem 2 *At any time t, $E(\gamma_t)$ is increasing in λ.*

In accordance with Fact #1, our model predicts that the expected level of geographic concentration in an industry is increasing in the pace of incumbent innovation. The intuition is that more innovative industries provide more opportunities for spinoffs to form, and it is (only) spinoffs that give rise to clustering. (Alternatively, as we decrease the rate of innovation and the number of outside startups gets large relative to the number of spinoffs, clustering eventually vanishes.) It is not surprising that industries like semiconductors and automobiles exhibit the most extreme clustering. They are famously innovative industries that underwent rapid expansion, and with so many new ideas being pursued – as well as not being pursued – by incumbent firms, they naturally had opportunities for spinoffs and hence ended up highly clustered.

5 The Growth of a Cluster

Having established in Section 4 that clustering will occur in our model, especially in innovative industries, we now shift our focus to the origins of a cluster in a particular region. Recall that Facts #2-5 tell a story of large business clusters prominently featuring spinoffs. Two main results in this section, Theorems 3 and 4, show what appears to be a virtuous

cycle with spinoffs and regional growth. A region's market share is predictive of the birth of spinoffs, and spinoffs are predictive of a region's growth. These results directly address Facts #3 and #5 respectively. Underlying both phenomena, according to our model, is a process of innovation with inherent positive feedback. New innovations build on previous innovations, and they lead to spinoffs and to growth. This self-reinforcing dynamic can over time amplify an initial advantage in one particular region, an implication (Theorem 5) that conforms with Fact #2. We will defer discussion of Fact #4 to Section 6, after we have presented results about the correlates of successful spinoffs. We will come to see this fact as a natural consequence of a few other stylized facts in our list.

After recognizing that semiconductors and autos are natural candidates for clustering, we might still ask, why Silicon Valley or why Detroit? How does a particular region come to lead an industry? The key ingredient, according to our model, is innovation, and with innovation comes spinoffs. Spinoffs should then be more common in highly active regions:

Theorem 3 *For any region r and times $t'' > t' \geq t$,*

$$\operatorname{cov}\left(\varpi_{r,t},\, \varsigma_r(t', t'')\right) > 0.$$

Theorem 3 states that there is a positive correlation between a region's share of the profits in the industry at a given time and the number of spinoffs subsequently spawned there. Intuitively, regions with greater market share are expected to generate more innovations and hence more spinoffs. On the other hand, for any particular region the rate of entry of outside startups is independent of the market share of the region. We thus have a very straightforward account of the stylized fact that a greater percentage of the entrants in clusters than elsewhere are spinoffs (Fact #3).

We have accounted for the pattern of spinoffs springing up in clusters, but, in addition, we observe spinoffs playing a dominant role in the growth of a cluster. In order to charac-

terize a region's growth in our model, we introduce a bit of notation. Let $\Delta\pi_{j,t,\Delta t}$ denote the (profit) growth of firm j during the interval $(t, t+\Delta t)$.[13] Similarly, let $\Delta\varpi_{r,t,\Delta t}$ denote the growth of region r during the interval $(t, t+\Delta t)$.[14]

It is straightforward to see that firms' ability to innovate on each of the submarkets they have already entered, as well as the correlation in profits between related submarkets, means that larger firms tend to grow more in absolute terms. Firm growth is approximately, but not precisely, in accordance with Gibrat's law, i.e., proportional to current size.[15] Similarly, by aggregation of all firms in a region, larger regions also tend to grow more in absolute terms. (See Proposition 1 in the appendix.) This means we often want to control for the current size of a region when examining the absolute growth of a region.

Even after controlling for the current size of a region, the number of spinoffs in the region should in our model correlate with the region's subsequent growth. We can establish this result formally in the case that there is no heterogeneity in the quality parameters (i.e., no uncertainty about the quality of each of the attributes discovered in the region). Let $\mathcal{X}_{r,t} = \bigcup_{j:\, u_{j,r}=1} X_{j,t}$ denote the set of submarkets that firms in region r have entered at time t.

Theorem 4 *For any region r, time t and interval $\Delta t > 0$, and any quality z (that we shall*

[13] We can express the expected growth of firm j('s profits) at time t, conditional on the set of submarkets the firm has already entered, as

$$\dot{\pi}_{j,t} \equiv \lim_{\Delta t \to 0} E\left(\Delta\pi_{j,t,\Delta t} \mid X_{j,t}\right) = \sum_{x \in X_{j,t}} \lambda(1-\alpha)m \int_S E\left(\left(\underline{\eta}\left(x \cup \{s\}\right)\right)^2\right) \mu(ds). \tag{2}$$

[14] The expected growth at time t of region r('s profits) is

$$\dot{\varpi}_{r,t} \equiv \lim_{\Delta t \to 0} E\left(\Delta\varpi_{r,t,\Delta t} \mid \mathcal{X}_{r,t}\right) = \kappa(t)\, f_r m \int_S E\left(\left(\underline{\eta}\left(\{s\}\right)\right)^2\right) \mu(ds) \,+\, \sum_j u_{j,r}\, \frac{1}{1-\alpha}\, \dot{\pi}_{j,t},$$

capturing the internal growth of existing firms as well as growth through the entry of new firms into the region.

[15] Firms pursue new submarkets in proportion to their current number of submarkets, but the profits generated in each submarket, while positively correlated within a firm or within a region, experience some reversion to the mean as well. This means that the relative growth rate of larger firms is less than that of smaller firms.

assume characterizes all attributes discovered in the region) satisfying $\text{Prob}(\underline{s} \in S_z) > 0$*,*

$$\text{cov}\left(\varsigma_r(0,t),\, \Delta\varpi_{r,t,\Delta t} \;\middle|\; \varpi_{r,t},\, \bigcup \mathcal{X}_{r,t} \subset S_z\right) > 0.$$

In Theorem 4 we are conditioning on the quality of the attributes discovered by firms in the region, but not on the identity of these attributes, by restricting innovations to a subset of attributes with homogeneous quality. The theorem states that conditional on this homogeneous attribute quality and on a region's total profit, there is a positive correlation between the number of spinoffs that have formed in the region and the region's subsequent growth in profits. This conforms with Fact #5, which indicates that spinoffs stimulate the growth of highly clustered regions.

Intuitively, comparing two regions with the same total profits, the one with more past spinoffs can be expected to have discovered more new submarkets. Assuming no heterogeneity in the quality parameters allows us to fully attribute the success of the region with less spinoffs to a high (random) realization of demand as opposed to high quality of innovations discovered there in the past. The latter would bear on the future growth of the region (through the quality of the submarkets subsequently discovered there), which can now be ruled out. So, the region with more spinoffs would typically continue to be more innovative and to experience more subsequent growth.

The assumption of homogeneous quality simplifies our analysis, but is not a necessary condition. The higher realizations of demand in the region with fewer spinoffs signal higher quality parameters, but the relative lack of spinoffs itself signals the contrary. The net signal is equivocal. Moreover, newly discovered submarkets will gradually diverge from existing ones, so the quality of past innovations matters less over time. On the other hand, the greater pace of innovation in the region with more spinoffs continues to reinforce itself.

With this insight we can return to the exemplary Silicon Valley semiconductor cluster

and address the question of what gives rise to such powerful growth. In particular, why did Silicon Valley grow faster than Dallas when both regions once had similar market shares and successful flagship firms? Theorem 4 suggests a particularly simple account: Silicon Valley had more spinoffs. Merely splitting off economic activity into separate corporate entities might not fundamentally cause explosive growth, but it might well indicate particularly fertile ground for innovation and hence for explosive growth.[16] Both Fairchild in Silicon Valley and Texas Instruments in Dallas discovered rich veins in which to innovate, openings into submarkets which would turn out to be immensely profitable. Yet perhaps the entry of so many spinoffs from Fairchild suggests that these firms had more innovative opportunities, that they were better positioned to discover the next great innovations that would drive the growth of Silicon Valley ahead of Dallas.

We have shown that in our model spinoffs accompany the growth of a cluster in a virtuous cycle. Discovery of new submarkets is at the heart of the spinoff process just as it also pumps up the growth of a region. To see how this process might get started, we apply Theorem 3 to the case of a region gaining its first firm in the industry. The size (profits) upon entry of the initial entrant in a region is predictive of the number of spinoffs subsequently spawned in that region. Letting t_j^0 be the time at which firm j formed (and $t_{\hat{\jmath}_r}^0$ specifically be the first time a firm formed in region r):

Theorem 5 *For any region r and times $t'' > t' \geq t_{\hat{\jmath}_r}^0$,*

$$\operatorname{cov}\left(\pi_{\hat{\jmath}_r, t_{\hat{\jmath}_r}^0},\, \varsigma_r(t', t'')\right) > 0.$$

Theorem 5 states that there is positive correlation between the profit upon entry of the

[16] Consistent with this account, regions with many small firms (alongside at least one big firm) are more innovative (i.e., produce more cited patents) and have a higher rate of spinoff formation than regions without so many small firms, even after controlling for the size of (i.e., the number of inventors in) each region (Agrawal et al. [2014]).

initial firm in a region and the number of spinoffs subsequently spawned in the region. This means that regions with flagship firms (industry leading initial entrants) are expected to have more spinoff entrants. The profit of the initial entrant is a signal of the quality of the first attribute discovered in the region. This attribute will influence the quality of future submarkets discovered in the region, which in turn conditions whether potential spinoffs will actually form after an incumbent (potential parent) firm does not pursue a new submarket. Theorem 5 thus provides an account for the connection of flagship firms in a region and the subsequent number of spinoffs there (Fact #2).

6 Spinoff Entry and Performance

Sections 4 and 5 have shown how our model accounts for the remarkable connection between booming industry clusters and spinoffs. Still, it remains to show that our model accords with the observed patterns about spinoff formation and performance, as described by Facts #6-10, as well as Fact #4. We now derive results that illustrate these patterns as they arise in our model.

Some additional notation will help us present these results. Let $\underline{j}_t$ be a randomly selected firm drawn uniformly from among all firms in existence at time t, i.e., from $\{1, \ldots, J_t\}$. When we don't care about the particular time t at which the draw is made, we may neglect the subscript. Conditioning on the type of firm selected, we identify a randomly selected outside startup as $\underline{j}^{\mathsf{o}}$ and a randomly selected spinoff firm as $\underline{j}^{\mathsf{s}}$.

We begin with the pattern that spinoffs on average tend to be better performers (i.e., more profitable at every age) than other entrants. For this particular result, we adopt the specific functional form of the mixed geometric distribution for a submarket's output along with the restriction that most innovations fail. Our model then implies that spinoffs do better than outside startups.

Theorem 6 *Assume that for any submarket x, output $\underline{\eta}(x)$ has a mixed geometric distribu-*

tion, and restrict $z_s \in (0, \frac{1}{2}]$ *for all* s*. At any age* τ*,*

$$E\Big(\pi_{\underline{j}^s,\,\tau+t^0_{\underline{j}^s}}\Big) > E\Big(\pi_{\underline{j}^o,\,\tau+t^0_{\underline{j}^o}}\Big).$$

In accordance with Fact #6, our model predicts that spinoffs tend to be more profitable at any given age than outside startups at the same age. The intuition is that spinoffs are more likely to enter near a higher performing segment of submarkets. The fact that a firm is a spinoff instead of an outside startup carries information. The spinoff necessarily enters the industry in a submarket similar (in terms of shared attributes) to some other submarket that has already proven to be successful for its parent. Because these attributes are incorporated into the innovations the spinoff subsequently pursues, demand tends to be higher in the submarkets the spinoff enters than in randomly discovered submarkets stemming from an entry-level submarket. Consequently, the spinoff is expected to be more profitable than an outside startup.

Our model also predicts that firms with greater profits (or, similarly, greater market share) tend to spawn more spinoffs.

Theorem 7 *For times* $t'' > t' \geq t$ *and any firm* j *selected at time* t*,*

$$\operatorname{cov}\left(\pi_{j,t},\, \sigma_j(t', t'')\right) > 0.$$

Theorem 7 states that there is a positive correlation between firm profits at a given time and the number of spinoffs that the firm subsequently spawns. This result is consistent with Fact #7. Intuitively, expanding into more submarkets adds to profits and also creates more opportunities for discovering innovations that occasionally lead to spinoffs. Additionally, finding high quality submarkets both increases current profits and also increases the likelihood that future innovations will be successful, thereby further enabling spinoff entry. As

profits follow revenue and, more fundamentally, output, the same argument could be used to show a positive correlation between market share and subsequent spinoffs.

We can identify multiple factors that influence the quality of our spinoffs. Our model predicts that more successful spinoffs tend to have more successful parents. Additionally, spinoffs that are initially more successful tend to do better subsequently. Letting $\rho(j^{\mathfrak{s}})$ denote the parent firm from which the spinoff $j^{\mathfrak{s}}$ formed, we have:

Theorem 8 *For any spinoff $j^{\mathfrak{s}}$, profit upon entry correlates with its parent's profit:*

$$\text{COV}\left(\pi_{j^{\mathfrak{s}},t^0_{j^{\mathfrak{s}}}},\ \pi_{\rho(j^{\mathfrak{s}}),t^0_{j^{\mathfrak{s}}}}\right) > 0.$$

Moreover, both profit upon entry and parent's profit are predictive of a spinoff's subsequent profit growth.[17] *For any firm age τ, and over any time span $\Delta t > 0$, we have*

$$\text{COV}\left(\Delta\pi_{j^{\mathfrak{s}},\tau+t^0_{j^{\mathfrak{s}}},\Delta t},\ \pi_{j^{\mathfrak{s}},t^0_{j^{\mathfrak{s}}}}\mid \pi_{\rho(j^{\mathfrak{s}}),t^0_{j^{\mathfrak{s}}}}\right) > 0$$

and

$$\text{COV}\left(\Delta\pi_{j^{\mathfrak{s}},\tau+t^0_{j^{\mathfrak{s}}},\Delta t},\ \pi_{\rho(j^{\mathfrak{s}}),t^0_{j^{\mathfrak{s}}}}\mid \pi_{j^{\mathfrak{s}},t^0_{j^{\mathfrak{s}}}}\right) > 0.$$

Theorem 8 indicates a positive correlation between a spinoff's profit upon entry and its parent firm's profit at that time, as well as between a spinoff's profit growth over time and both its profit upon entry and its parent's profit at that time, even after controlling for the other factor. The same correlations exist with revenues in place of profits. The correlation between a spinoff's performance and its parent's accords with Fact #8. The additional correlation with the spinoff's initial performance upon entry accords with Fact #9. Intuitively, because spinoffs enter the industry producing in submarkets that are related

[17]The positive correlation between profit growth and profit upon entry actually extends to all firms, not just to spinoffs.

to their parents' (i.e., they initially enter a submarket that shares all but one of its attributes with one of the parent's submarkets), the demand in their submarkets are correlated, so we find that better-performing parent firms breed better-performing spinoffs. On top of this, starting with high initial profit is another good omen because it too signals that the firm may have found a rich vein in which to innovate, i.e., that attributes that are retained in all their future innovations may have high quality, and thus demand in subsequently discovered submarkets is more likely to be strong.

Moreover, even after controlling for the size (profits) of the parent, the size (profits) of the entire region should still be predictive of the profitability of a new spinoff. The same reasoning as in Theorem 8 applies here as well. The success of the entire region provides another informative signal about the quality of the attributes that (partially) characterize the submarkets that the spinoff will enter.

Theorem 9 *For any spinoff $j^{\mathfrak{s}}$ in any region r (i.e., with $u_{j^{\mathfrak{s}},r} = 1$) and any time $t' > t^0_{j^{\mathfrak{s}}}$,*

$$\lim_{t \nearrow t^0_{j^{\mathfrak{s}}}} \operatorname{cov}\left(\varpi_{r,t},\ \pi_{j^{\mathfrak{s}},t'} \mid \pi_{\rho(j^{\mathfrak{s}}),\, t^0_{j^{\mathfrak{s}}}}\right) > 0.$$

Theorem 9 states that conditional on the parent's profit at the time it spawns a spinoff, there is still positive correlation between the total profit in the region at that time and the spinoff's profit subsequently. Looking to the extremely successful regions once again, we find that new spinoffs tend to be most successful in the leading (most highly clustered) region, as Fact #10 indicated.

Finally, we may return to Fact #4. The general pattern that spinoffs disproportionately became the industry leaders in the largest clusters follows naturally from two results already established: spinoffs generally outperform other entrants (Theorem 6), and spinoffs in these regions in particular have the most success (Theorem 9). When combined with Fact #3, that the leading regions also have had more spinoffs, it is perhaps no surprise that in some of the

industries that have been studied, the top tier of leaders eventually consisted exclusively of spinoffs.

7 The Extent of Clustering

Sections 4-6 have established that our model of the process of innovation and spinoff formation yields a positive expected level of geographic concentration and accords with the eleven stylized facts described in Section 2. Still, for practical application, we would like to know how much geographic concentration can be attributed to the innovation and spinoff process so we can obtain better estimates of the role that traditional localized externalities play over and above it.

Theorem 1 in Section 4 tells us precisely how much geographic concentration can be attributed to the innovation and spinoff process and even suggests a straightforward correction to the Ellison-Glaeser index that would measure the extent of geographic concentration over and above this level: simply pool the market shares of all firms with shared heritage in each region, as if these firms composed a single unit, when computing the Herfindahl index, i.e., replace H_t with $\hat{H}_t$. Application of this result requires us to trace the industry's entire heritage. In practice, tracking the organizational heritage of all the firms in an industry takes a lot of hard work and has only been done for a few select industries. Until researchers collect this empirical data, we find it useful to make some numerical estimates of the degree of clustering that arises in our model solely from the innovation and spinoff process, in the absence of localized externalities.

Each panel of Figure 1 shows the Ellison-Glaeser index of geographic concentration over time from 100 simulated runs of the model[18] with reasonable parameter values.[19] Each

[18] Our simulation takes the regions to be the 50 states plus the District of Columbia ($R = 51$) and takes the share of total economic activity in each region f_r to be the share of manufacturing employment there, using December 2013 BLS data from www.bls.gov.

[19] The particular specification assumes a constant arrival rate of outside startups, $\kappa(t) = 1$, a mixed geometric distribution for the output $\underline{\eta}(x)$ in submarket x, and a uniform distribution over $[0, \frac{1}{2}]$ for the quality of a randomly discovered attribute $z_{\underline{s}}$.

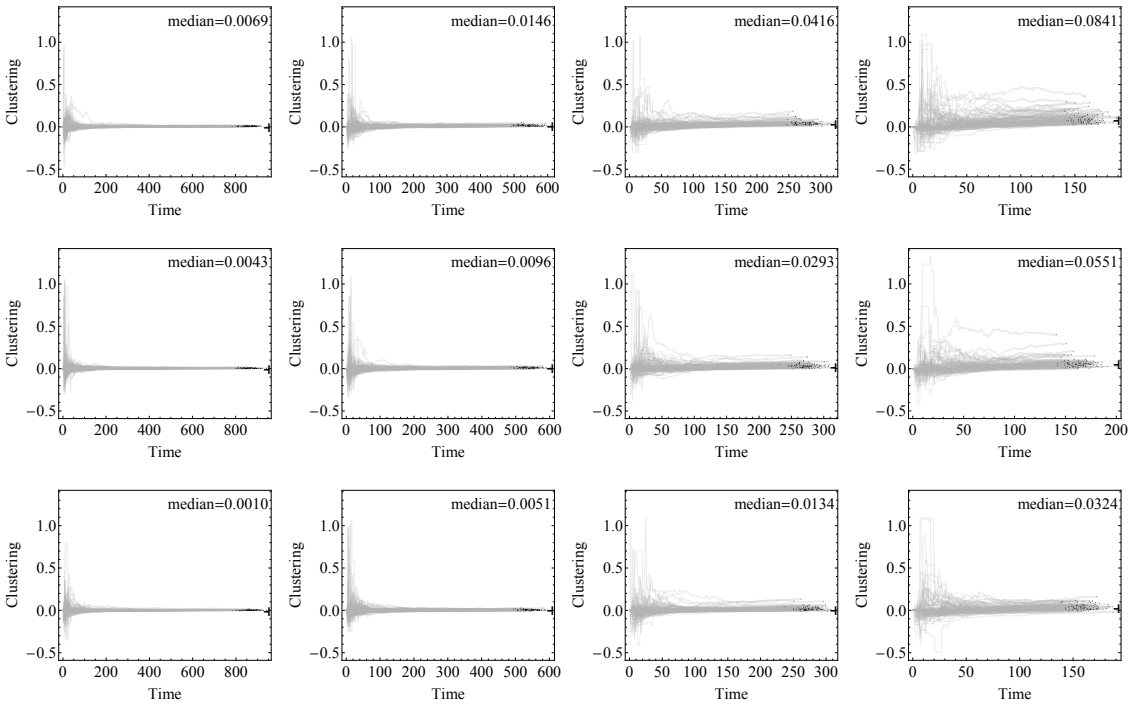

Figure 1: The Ellison-Glaeser index of geographic concentration over time for 100 simulated runs of the model, with each run terminating upon reaching 1000 submarkets. The innovation rate λ varies across the columns, taking on values 0.01, 0.02, 0.05, and 0.1 from left to right. The splitting probability α varies across the rows, taking on values 0.05, 0.1, and 0.2 from bottom to top.

simulation was run until there were 1000 submarkets. The time required to reach this many submarkets varies between runs, but in all cases the long-run behavior of the index becomes clear by this point. The Ellison-Glaeser index appears to converge asymptotically to a level that is path dependent. Remarkably, it does not decay toward 0 nor explode toward 1. Two opposing forces are in balance. Over time the pace of spinoff entry picks up, and that causes clustering. However, the effect can be seen as distorting the correction for the finite sample of firms, and the number of firms grows large over time, so this correction would fade if not for the spinoff and innovation dynamics. If the largest family of firms were to occupy merely an infinitesimal share of the industry as the number of families grew large, then the correction would fade entirely and the index would decay toward 0. However, the positive feedback in the innovation process implies that larger families of firms grow more quickly and do not ever become inconsequential.[20] The population of firms does not inevitably spread out evenly across locations.[21] Thus, clustering can persist.

For each specification of the innovation rate λ (the column) and the splitting probability α (the row), the median asymptotic value of the Ellison-Glaeser index (estimated when reaching 1000 submarkets) is noted in Figure 1. As Theorem 2 claims, the index is increasing in λ. We also see that the index is increasing in α, consistent with the intuition that more spinoff formation makes an industry appear more highly clustered. If the pace of innovation is too slow ($\lambda = .01$) and splitting from one's parent firm is relatively unlikely ($\alpha = .05$), then clustering practically vanishes. At the other extreme, with rapid innovation ($\lambda = .1$) and a relatively high probability of splitting from one's parent firm ($\alpha = .2$), then the index is atypically large (median value 0.084 and topping out near 0.4 in the largest

[20] Bottazzi and Secchi (2006) and Luttmer (2007) present models of firm growth in which new technologies or business opportunities grow out of existing ones, and this positive feedback causes the firm size distribution to have a fat tail. A similar mechanism in our model is the reason why the largest family of firms retains a non-negligible market share.

[21] The population of families of firms does spread out proportionally across locations in the long run, but the largest families continue to account for a disproportionate share of the industry.

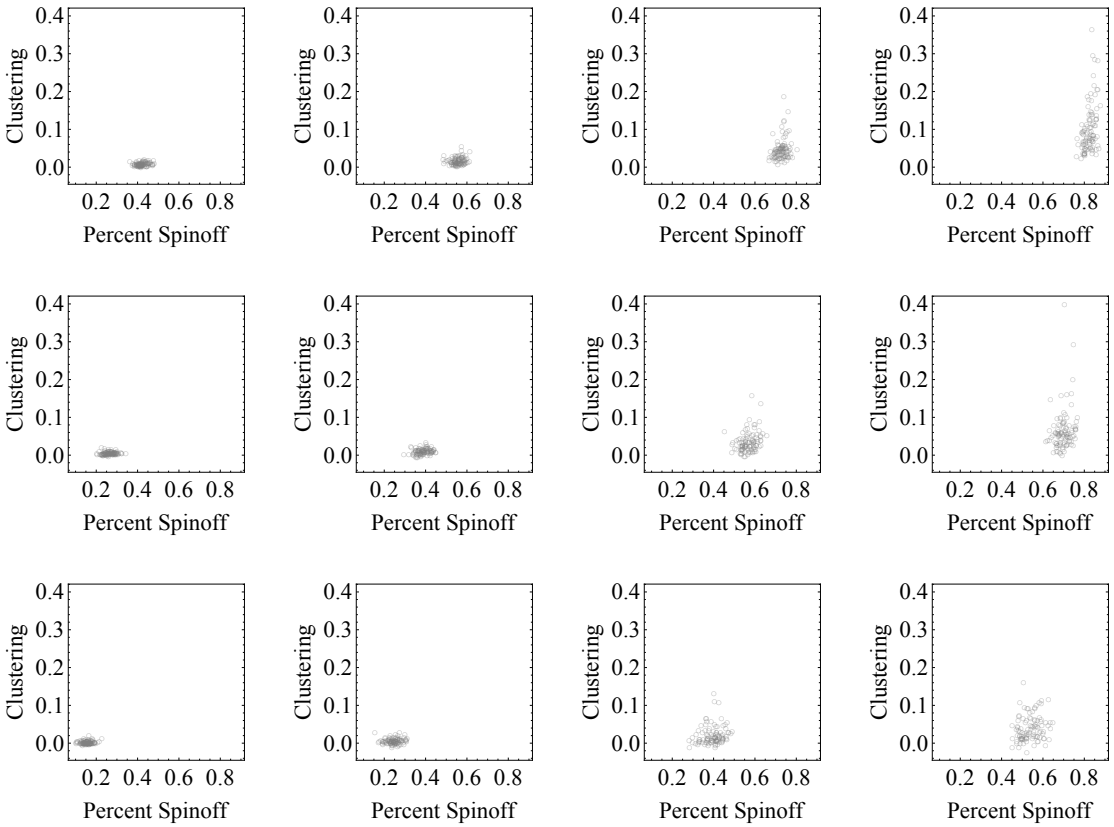

Figure 2: The Ellison-Glaeser index of geographic concentration and the percentage of firms that are spinoffs, recorded upon reaching 1000 submarkets, across all 100 simulated runs of the model. The innovation rate λ varies across the columns, taking on values 0.01, 0.02, 0.05, and 0.1 from left to right. The splitting probability α varies across the rows, taking on values 0.05, 0.1, and 0.2 from bottom to top.

of the 100 simulated runs). For comparison, Ellison and Glaeser (1997) report the median value of their index across 459 U.S. manufacturing industries as 0.026, along with mean value 0.051, and they identify only four industries with an index greater than 0.4. We do not have much basis for a precise specification of realistic parameter values (which may well vary across industries) in our model, but we might use $\lambda = .02$ and $\alpha = .1$ to make a point estimate of 0.010 as the typical level of the Ellison-Glaeser index with our model. This estimate suggests that the dynamics of innovation and spinoffs might account for a large part, but not all, of the observed geographic concentration of manufacturing industries.

Figure 2 shows scatter plots of the Ellison-Glaeser index and the percentage of firms that are spinoffs at the termination of each run, upon reaching 1000 submarkets. A straightforward consequence of spinoff formation requiring both innovation and splitting off of this activity is that the percentage of spinoffs (just like the Ellison-Glaeser index) is increasing in the innovation rate λ (moving to the right across the columns) and in the splitting probability α (moving up the rows). Additionally, there is some degree of correlation between the percentage of spinoffs and the Ellison-Glaeser index within each scatter plot, but the variation within a plot is less pronounced than the variation between plots. This cross-sectional view shows that more spinoffs lead to greater clustering. The precise value of the percentage of spinoffs, however, depends on our arbitrary termination point. If the process were to run forever, the population of firms would come to be completely dominated by spinoffs. This clearly unrealistic implication of our model is a result of the absence here of any shakeout process, which surely does operate in the real world.[22]

Recall that the Ellison-Glaeser index is designed to control for the lumpiness of firms so that if each firm's location were independent of its size, the index would not depend on the Herfindahl. A distinguishing feature of our model is that firms in the same region are correlated in size (because of the possibility of shared heritage), so the Ellison-Glaeser index does correlate with the Herfindahl index (as shown in Figure 3 in the appendix).

8 Discussion and Conclusion

The empirical pattern of clusters in various innovative industries growing principally through spinoff entry – and, moreover, of these spinoffs coming to dominate their industries – is striking and calls out for explanation. Traditional economic theory is mostly silent on the origins of a firm. In passing, standard theory might at best suggest that entry is more likely

[22] Allowing submarkets to perish with an exogenous hazard rate would let us address patterns of firm exit and perhaps capture industry shakeouts as well, as in Klepper and Thompson [2006]. Spinoffs' longer survival could possibly account for the higher survival rate in clusters (cf. Dumais et al. [2002]).

to take place in clusters because of the advantages of clustering. Perhaps entrants founded by individuals leaving incumbent firms in the industry somehow have a unique ability to exploit these advantages. In essence, conventional wisdom attributes observed patterns of clustering to firms' incentives. We take a different perspective and suggest that opportunity matters as much as incentives.[23] Innovation creates the opportunity for spinoff firms to form and for an industry cluster to grow (Schumpeter [1934, 1942]). We develop a theory based on an underlying process of innovation with positive feedback to account for the close connection between spinoffs and clustering. Our theory explains empirical regularities about the clustering of innovative industries, the growth of a cluster in a particular region where spinoffs have proliferated, and the entry and performance of these spinoffs.

Clusters emerge in our model even in the absence of agglomeration economies. Obviously, the presence of other firms in the same industry can affect the economic climate in a particular region. Our model, however, cautions against jumping from the observation of pervasive industry clustering to the conclusion that powerful agglomeration economies are universal. We noted in Section 1 that firms in clusters perform better, that entry is concentrated in these regions, and that firms in related industries tend to locate near each other as well. Each of these patterns could be driven exclusively by spinoffs, which happen to spring up where there is already activity in the industry and which then outperform other firms. (To explain the third pattern, we could allow innovations to occasionally cross the boundaries between industries.) Indeed, in many cases the superior performance of firms in clusters does not extend beyond spinoffs. In the U.S. automobile industry (Klepper [2007]), the tire industry (Buenstorf and Klepper [2009, 2010]), the Dutch book publishing industry (Heebels and Boschma [2011]), the fashion design industry (Wenting [2008]), and

[23]These perspectives really are complements, not substitutes. Incentives and opportunity both, no doubt, shape firm behavior. So, too, rational allocation of capital (i.e., responsiveness to incentives) and discovery of innovation (i.e., exploitation of opportunity) are both necessary ingredients for economic growth. We merely suggest that the latter constraint is sometimes binding.

the British automobile industry (Boschma and Wenting [2007]), spinoffs in the respective industry clusters tend to be superior performers, but other new entrants inside industry clusters have comparable performance to other entrants elsewhere.[24] A survey across industries shows strong evidence that clusters promote entry, but little evidence that they enhance firm growth or survival (Frenken et al. [2015]).[25]

The evidence (e.g., higher spinoff rates inside clusters (even after controlling for firm characteristics)) and our model are consistent with a view of knowledge spillovers, traditionally thought to benefit an entire cluster, as a Marshallian externality that is specific to entrepreneurship.[26] These spillovers primarily support the formation of spinoffs, which have access to the technical knowledge developed by their parents. Tacit knowledge facilitates innovation, and such knowledge is difficult to acquire without being inside an organization that possesses it. Knowledge can be transferred both through founders and employees that entrants hire from incumbent firms (Breschi and Lissoni [2009], Cheyre et al. [2015]). A desire to hire employees from their parents, to tap into their specialized knowledge base, may well be an important motive for spinoffs to locate close to their parents (Carias and Klepper [2010]), which is of course the critical ingredient for clustering in our model.[27]

The phenomenon of spinoffs locating near their parents can occur on top of more broadly based agglomeration economies or natural advantages that favor a particular region. Such complementary forces surely do contribute to the agglomeration of at least a few notable industries (e.g., the movie industry in Hollywood, finance in New York City,

[24] A conspicuous absence of positive externalities in clusters has also been noted in the metal-working (Appold [1995]), footwear (Sorenson and Audia [2000]), knitwear (Staber [2001]), biotechnology (Stuart and Sorenson [2003]), and machine tool (Buenstorf and Guenther [2011]) industries.

[25] These null findings are consistent with the observation that the development of economic institutions in a region tends to lag firm growth rather than to precipitate it (Feldman [2001]).

[26] There could also be a demonstration effect encouraging entrepreneurship (Nanda and Sørensen [2010]).

[27] The notion that spinoffs might in their formative stages need to recruit labor from their parents helps us understand why congestion costs in clusters might persist and not drive spinoffs away. (See Stam [2007] for a careful treatment of relocation costs.)

steel in Pittsburgh and wine in northern California) and our calculations of the extent of geographic concentration showing up in our model in Section 7 reveal that the model has room for them. It would be straightforward to accommodate agglomeration economies and natural regional advantages in our model by conditioning $\underline{\eta}(x)$ (i.e., the firm's output in a given submarket and, in turn, the firm's profit in that submarket) on the location of the firm as well as the level of industry activity in the region.[28] This would generate even more concentrated regional clusters and a stronger correlation between the market share of a region and the success of a new firm there. This correlation would then extend beyond spinoffs to all firms entering the region, but would still be stronger for spinoffs than for other entrants. Additionally, this extension of the model would not interfere with, and actually would strengthen some of, the other patterns we have described.

Our model of the dynamics of spinoff formation and clustering fits naturally into a framework for evolutionary economic geography that conceives of innovation as a branching process that generates industrial and urban growth (Boschma and Frenken [2006], Frenken and Boschma [2007], Buendia [2013]). Economic development is a complex system. Our model formalizes the intuition that clusters form endogenously, driven by entrepreneurs building their own firms (Feldman et al. [2005]). Firms grow over time, discovering new submarkets through innovation. Occasionally spinoffs form to pursue these opportunities. We suggest that in some cases clusters arise as an artifact of the spinoff process, rather than as the basis for it. Nevertheless, business clusters are still indicative of rapid technological change and industrial growth.

[28] Conditioning a firm's output on its location would be equivalent to assuming that all submarkets include an attribute characterizing their locations. It would be as if all firms in each region were related.

Appendix

Glossary of Mathematical Notation

x	a submarket, characterized by a finite set of attributes
$\|x\|$	the number of attributes that characterize submarket x
$\underline{s}$	a randomly discovered attribute
$X_{j,t}$	the set of submarkets that firm j has entered at time t
$N_{j,t}$	the number of submarkets that firm j has entered at time t
λ	the rate of innovation in each existing submarket
$\underline{\eta}(x)$	the output in submarket x
$\pi_{j,t}$	the profit firm j earns at time t
z_s	the quality of attribute s
$\bar{z}_x$	the average quality of the attributes of submarket x
S_z	the set of attributes having quality z
α	the probability that an incumbent firm does not pursue a submarket it discovers and instead a spinoff attempts to form to pursue it
$\sigma_j(t,t')$	the number of spinoffs that come out of firm j during the interval (t,t')
$\kappa(t)$	the mean arrival rate of outside startups at time t
f_r	the fraction of overall economic activity occurring in region r
$u_{j,r}$	an indicator variable for firm j being located in region r
J_t	the number of firms in the industry at time t
$\varpi_{r,t}$	the total industry profits in region r at time t
$\varsigma_r(t,t')$	the number of spinoffs that form in region r during the interval (t,t')
$\hat{\jmath}_r$	the first entrant in region r
$\theta_{j,t}$	the market share of firm j at time t
$\vartheta_{r,t}$	the combined market share of all firms in region r at time t
H_t	the industry Herfindahl index at time t
γ_t	the Ellison-Glaeser index of geographic concentration at time t
$\Delta\pi_{j,t,\Delta t}$	firm j's profit growth during the interval $(t, t+\Delta t)$
$\Delta\varpi_{r,t,\Delta t}$	the growth of region r during the interval $(t, t+\Delta t)$
$\mathcal{X}_{r,t}$	the set of submarkets that some firm in region r has entered at time t
$\mathcal{N}_{r,t}$	the number of submarkets that firms in region r have entered at time t
t^0_j	the time at which firm j formed
$x^0_{j^{\mathfrak{s}}}$	the first submarket entered by spinoff $j^{\mathfrak{s}}$
$\underline{j}^{\mathfrak{o}}$	a randomly selected outside startup
$\underline{j}^{\mathfrak{s}}$	a randomly selected spinoff firm
$\rho(j^{\mathfrak{s}})$	the parent of spinoff $j^{\mathfrak{s}}$
$\beta(j^{\mathfrak{o}})$	the family of firms descending from the startup $j^{\mathfrak{o}}$

Mathematical Proofs

We begin by introducing a mathematical result that will be useful in many proofs below.

Lemma 2 *Let A, B, and Y be random variables. If A and B are conditionally independent given Y and their conditional expectations are increasing with Y, then* $\mathrm{cov}(A, B) > 0$. *More generally, we can let Y be a set of random variables $Y_1, \ldots, Y_n$, and it suffices to assume (along with conditional independence) that $E(A|Y_1, \ldots, Y_i)$ and $E(B|Y_1, \ldots, Y_i)$ are increasing in Y_i for all i.*

Proof We begin with the special case that Y is just a single variable. The law of total covariance states that

$$\mathrm{cov}(A, B) = E\big(\mathrm{cov}(A, B \,|\, Y)\big) + \mathrm{cov}\big(E(A|Y),\, E(B|Y)\big).$$

Conditional independence implies that $\mathrm{cov}(A, B \,|\, Y) = 0$, so the first term vanishes. To show that the second term is positive, we introduce the functions $g_A(Y) = E(A|Y)$ and $g_B(Y) = E(B|Y)$ and an iid copy of the random variable $Y' \sim Y$. Whenever Y and Y' take on different values y and y' respectively, we have $\big(g_A(y) - g_A(y')\big)\big(g_B(y) - g_B(y')\big) > 0$, because g_A and g_B are assumed to be increasing, so both factors are positive when $y > y'$ and both factors are negative when $y < y'$. Taking expectations with respect to Y and Y', we get:

$$E\big(g_A(Y)g_B(Y)\big) - E\big(g_A(Y)\big)E\big(g_B(Y')\big) - E\big(g_A(Y')\big)E\big(g_B(Y)\big) + E\big(g_A(Y')g_B(Y')\big) > 0.$$

So, indeed, $\mathrm{cov}\big(g_A(Y), g_B(Y)\big) > 0$.

For the generalization, we need only iterate the law of total covariance by first condi-

tioning on Y_1 and then successively on each subsequent Y_i. We obtain

$$\mathrm{cov}(A, B) = E\big(\mathrm{cov}(A, B \mid Y)\big) + \sum_{i=1}^{n} E\Big(\mathrm{cov}\big(E_{Y_i}(A|Y_1, \ldots, Y_i),\ E_{Y_i}(B|Y_1, \ldots, Y_i)\big)\Big).$$

Once again, the first term vanishes due to conditional independence, and the remaining terms are positive due to the same argument as in the special case. (For the i'th term, the expectation is taken over $Y_1, \ldots, Y_{i-1}$, and the covariance in that term is positive regardless of the value taken by these random variables, so the expectation must be positive.) ∎

Proof of Lemma 1

To simplify notation, drop the time t subscript, and consider the market shares $\theta_{j,t}$ to be fixed so that conditioning on them is implicit without the need to carry them around. Expand the quadratic in the definition of L and use the linearity of the expectation operator to write

$$E(L) = \sum_{r} E(\vartheta_r^2) - 2f_r E(\vartheta_r) + f_r^2. \tag{3}$$

Using $\mathrm{Prob}(u_{j,r} = 1) = f_r$ and $\mathrm{Prob}(u_{j,r} = 0) = 1 - f_r$, which implies $\mathrm{Prob}(u_{j,r}^2 = 1) = f_r$ and $\mathrm{Prob}(u_{j,r}^2 = 0) = 1 - f_r$ and for $j \neq j'$, $\mathrm{Prob}(u_{j,r}u_{j',r} = 1) = f_r^2$ and

$\text{Prob}(u_{j,r}u_{j',r} = 0) = 1 - f_r^2$, we then obtain:

$$\begin{aligned}
E(\vartheta_r^2) &= E\left(\Big(\sum_j u_{j,r}\theta_j\Big)^2\right) \\
&= E\left(\sum_j u_{j,r}^2\theta_j^2\right) + E\left(\sum_{j\neq j'} u_{j,r}u_{j',r}\theta_j\theta_{j'}\right) \\
&= \sum_j \theta_j^2 E(u_{j,r}^2) + \sum_{j\neq j'} \theta_j\theta_{j'} E(u_{j,r}u_{j',r}) \\
&= \sum_j \theta_j^2 f_r + \sum_{j\neq j'} \theta_j\theta_{j'} f_r^2 \\
&= f_r H + \left(\Big(\sum_j \theta_j\Big)^2 - \sum_j \theta_j^2\right) f_r^2 \\
&= f_r H + f_r^2(1-H).
\end{aligned}$$

We can also calculate

$$\begin{aligned}
E(\vartheta_r) &= E(\sum_j u_{j,r}\theta_j) \\
&= \sum_j \theta_j E(u_{j,r}) \;=\; f_r.
\end{aligned}$$

Substituting into Equation (3), we then obtain

$$\begin{aligned}
E(L) &= \sum_r f_r H + f_r^2(1-H) - 2f_r^2 + f_r^2 \\
&= \sum_r (f_r - f_r^2)H \\
&= \left(1 - \sum_r f_r^2\right) H. \qquad \blacksquare
\end{aligned}$$

Proof of Theorem 1

It follows from Lemma 1 that

$$E\big(L_t \mid \big(\hat{\theta}_{j^o,t}\big)_{j^o\in\{1,\ldots,J_t\}}\big) = \left(1 - \sum_r f_r^2\right)\hat{H}_t.$$

We plug this directly into the definition of the Ellison-Glaeser clustering index γ_t. ∎

Proof of Corollary 1

We prove the stronger claim that for any firm-size distribution $\theta_{1,t}, \ldots, \theta_{J_t,t}$ with $J_t > 1$,

$$E\Big(\gamma_t \;\Big|\; J_t,\, \theta_{1,t}, \ldots, \theta_{J_t,t},\, \sum_r \varsigma_r(0,t)\Big) > 0 \text{ if and only if } \sum_r \varsigma_r(0,t) > 0.$$

We expand $\hat{\theta}_{j^o}$ (again omitting the t subscripts):

$$\hat{\theta}_{j^o}^2 = \Big(\sum_{j\in\beta(j^o)} \theta_j\Big)^2 = \sum_{j\in\beta(j^o)} \theta_j^2 + \sum_{\substack{j,j'\in\beta(j^o)\\ j\neq j'}} \theta_j\theta_{j'} \geq \sum_{j\in\beta(j^o)} \theta_j^2,$$

with the inequality strict whenever $|\beta(j^o)| > 1$, i.e., whenever $\sigma_{j^o}(0,t) > 0$ (because every $\theta_j > 0$.) This implies that conditional on $\sum_r \varsigma_r(0,t) > 0$, we know $\hat{H}_t > H_t$. ∎

Proof of Theorem 2

Applying the law of iterated expectations to the formula in Theorem 1, we have

$$E(\gamma_t) = E\left(\frac{\hat{H}_t - H_t}{1 - H_t}\right).$$

We would like to take the derivative with respect to λ and show that it is positive, but this derivative is not easily calculated. Instead we use a trick to show that it is positive, without ever expressing it in closed form. The trick is to consider how this derivative,

$\frac{\partial E(\gamma_t)}{\partial \lambda}$, depends on α.

If $\alpha = 0$ (and there were no spinoffs), then $E(\gamma_t) = 0$, and so, trivially, $E(\gamma_t)$ is constant with respect to λ, i.e., $\frac{\partial E(\gamma_t)}{\partial \lambda}\Big|_{\alpha=0} = 0$. If we find that $\frac{\partial E(\gamma_t)}{\partial \lambda}$ is increasing in α, then we will be able to conclude that it is always positive (for $\alpha > 0$). We now show that $\frac{\partial^2 E(\gamma_t)}{\partial \lambda \partial \alpha} > 0$.

Whether a submarket is pursued by an incumbent or a spinoff affects the distribution of market share among firms in the same family, but it does not affect the market share of the family itself. Thus, $\hat{H}_t$ does not vary with α. So,

$$\frac{\partial^2 E(\gamma_t)}{\partial \lambda \partial \alpha} = -\frac{\partial^2 E\left(\frac{H_t}{1-H_t}\right)}{\partial \lambda \partial \alpha},$$

and (because $\frac{H_t}{1-H_t}$ is a monotonic transformation of H_t) it remains to show that $\frac{\partial^2 E(H_t)}{\partial \lambda \partial \alpha} < 0$. A straightforward application of Jensen's inequality allows us to see that H_t decreases when the market share of a single firm is split into two separate firms (because $(\theta_{j^s} + \theta_{\rho(j^s)})^2 > \theta_{j^s}^2 + \theta_{\rho(j^s)}^2$), so $E(H_t)$ is decreasing in the fraction of innovations that are pursued by spinoffs, i.e., in α. The magnitude of this effect is amplified the more innovations there are, i.e., as λ increases, because each innovation presents another opportunity for a spinoff to form. So, indeed, $\frac{\partial^2 E(H_t)}{\partial \lambda \partial \alpha} < 0$. ∎

Proof of Theorem 3

This follows directly from Theorem 7. Aggregation across all firms in a region is straightforward. A region with a greater share of profits is composed of more profitable firms,[29] each of which tends to spawn more spinoffs. ∎

[29] By more profitable firms, we mean either firms that are more profitable or more firms that are profitable. The distinction is irrelevant.

Proof of Theorem 4

We use Lemma 2, conditioning on the number of submarkets in the region $\mathcal{N}_{r,t} \equiv |\mathcal{X}_{r,t}|$ (as well as on the quality parameters, though not the identities, of these submarkets). The number of spinoffs $\varsigma_r(0,t)$ in the region and the subsequent growth in profits $\Delta\varpi_{r,t,\Delta t}$ are conditionally independent because the former is determined by whether incumbent firms missed out on any of these submarkets and this has no bearing on the latter. It remains now to show that the conditional expectations of the number of spinoffs in the region and the subsequent profit growth there are both increasing in the number of submarkets discovered. Clearly the (conditional expectation of the) number of spinoffs is increasing in the number of submarkets discovered, because each non-entry-level submarket presents another opportunity for a spinoff to form. Similarly, the (conditional expectation of the) profit growth of the region is also increasing in the number of submarkets discovered, because each existing submarket presents another opportunity for an existing firm to innovate and expand the region.[30] ∎

Proof of Theorem 5

This follows directly from Theorem 3, letting $t = t^0_{\hat{\jmath}_r}$. ∎

Proof of Theorem 6

Consider a randomly chosen submarket $\underline{x}$ that an outside startup might discover. We argue based on symmetry that a spinoff $\underline{j}^{\mathfrak{s}}$ at the same age would be at least as likely to discover the submarket $\underline{x}'$ that contains the attributes in $\underline{x}$ as well as the attributes it inherited from its parent. Those attributes it inherits from the parent can be expected to be above average (because we can condition on the fact that the parent was able to form in the first place), so

[30] After conditioning on the region's current profits, the number of submarkets discovered is negatively correlated with the average profit of a submarket. Conditioning on the quality parameters ensures that this is independent of the profit from any subsequently discovered submarkets.

the outside startup's output $\underline{\eta}(\underline{x})$ would be stochastically dominated by the spinoff's output $\underline{\eta}(\underline{x}')$, too.

Formally, if $x_{j^{\mathfrak{s}}}^{-1}$ denotes the submarket upon which the parent $\rho(j^{\mathfrak{s}})$ innovated in the course of spawning firm $j^{\mathfrak{s}}$, then $\underline{x}' = x_{j^{\mathfrak{s}}}^{-1} \cup \underline{x}$. For any path by which the outside startup discovers submarket $\underline{x}$, the same path exists for the spinoff $j^{\mathfrak{s}}$ to discover $\underline{x}'$, i.e., the same attributes laid on top of a pre-existing innovation. Additionally, as discussed immediately below, the chance of such an innovation being successful is higher for the spinoff, which in turn leads to more opportunities to discover more innovations.

Our claim that $\underline{\eta}(\underline{x})$ is stochastically dominated by $\underline{\eta}(\underline{x}')$ (as well as the claim that the chance of success is higher for the spinoff) follows from the fact that $z_{\underline{s}} \mid \underline{s} \in \{x^{-1} : \underline{\eta}(x^{-1}) > 0\}$ stochastically dominates $z_{\underline{s}}$ (a convenient property of the mixed geometric distribution given the bound $z_{\underline{s}} \leq \frac{1}{2}$). That is, conditional on the success of submarket x^{-1}, and it must have been successful to allow a spinoff to form by innovating from it, the attributes that make up this submarket, and that are retained in submarkets the spinoff subsequently enters, should now be thought to have higher quality than an ordinary randomly discovered attribute. The higher quality of the submarkets the spinoff enters should translate to higher expected profits. ∎

Proof of Theorem 7

We use the general case of Lemma 2, conditioning on the set of submarkets firm j has entered, $X_{j,t}$. (First, condition on the number of submarkets, and then condition successively on each of the quality parameters.) Conditional on these submarkets, $\pi_{j,t}$ is independent of $\sigma_j(t', t'')$ because all that is left to influence the firm's profit is the realization of demand in each of these submarkets and, with the qualities of these submarkets fixed by the conditioning, this does not influence subsequent spinoff formation. We must now show that both of their conditional expectations are increasing in the realizations of the number of

submarkets $N_{j,t}$ and of the quality parameters z_s for all $s \in \bigcup X_{j,t}$.

1. It is clear that $E(\pi_{j,t} \mid X_{j,t})$ is increasing in $N_{j,t}$ and z_s for $s \in \bigcup X_{j,t}$ because $\pi_{j,t} = \sum_{x \in X_{j,t}} m \left(\underline{\eta}(x)\right)^2$ and $\underline{\eta}(x)$ is increasing (in the sense of stochastic dominance) in z_s for all $s \in x$. That is, the expected profit in any submarket is increasing in the quality parameters associated with the attributes of that submarket. Moreover, the profits in any particular submarket are non-negative, so increasing the number of submarkets can only increase expected total profit.

2. For every $\tilde{t} \in [t', t'']$, we can see that $\dot{\sigma}_{j,\tilde{t}}$ is increasing in $N_{j,t}$ and z_s for $s \in \bigcup X_{j,t}$ by examining Equation (1) (in Footnote 10). Increasing the number of submarkets at time t increases the opportunities for innovations leading to spinoffs at all future times, and increasing the quality parameters of attributes discovered by time t increases the likelihood of future innovations being successful, enabling more spinoff entry. ∎

Proof of Theorem 8

We use Lemma 2, conditioning on the first submarket entered by spinoff $j^{\mathfrak{s}}$, which we shall denote $x^0_{j^{\mathfrak{s}}}$. Conditional on (the quality parameters associated with) this submarket, the parent's profit $\pi_{\rho(j^{\mathfrak{s}}),\, t^0_{j^{\mathfrak{s}}}}$, the spinoff's initial profit $\pi_{j^{\mathfrak{s}},\, t^0_{j^{\mathfrak{s}}}}$, and the spinoff's subsequent growth $\Delta\pi_{j^{\mathfrak{s}},\, \tau + t^0_{j^{\mathfrak{s}}},\, \Delta t}$ are all independent. Again we show that each of these conditional expectations is increasing in the quality parameters associated with attributes of this first submarket.

1. There exists an attribute $s^* \in x^0_{j^{\mathfrak{s}}}$ (perhaps many of them) that was present in all of the parent firm's submarkets as well (perhaps along with additional attributes $\{s^{**}\}$ that may be present in many, but not necessarily all, of the parent firm's submarkets). The parent's expected conditional profit
   $E\left(\pi_{\rho(j^{\mathfrak{s}}),\, t^0_{j^{\mathfrak{s}}}} \mid x^0_{j^{\mathfrak{s}}}\right)$ is then increasing in z_{s^*} (as well as any other $z_{s^{**}}$).

2. The spinoff's expected conditional profit upon entry $E\big(\pi_{j^{\mathfrak{s}},\,t^0_{j^{\mathfrak{s}}}} \mid x^0_{j^{\mathfrak{s}}}\big)$ is obviously increasing in the quality parameters z_s for $s \in x^0_{j^{\mathfrak{s}}}$.

3. Noting that $x^0_{j^{\mathfrak{s}}} \subseteq x$ for all $x \in X_{j^{\mathfrak{s}},\tilde{t}}$, for $\tilde{t} \in [\tau + t^0_{j^{\mathfrak{s}}}, \tau + t^0_{j^{\mathfrak{s}}} + \Delta t]$, we can refer to Equation (2) (in Footnote 13) to see that $\dot{\pi}_{j^{\mathfrak{s}},\tilde{t}}$ is increasing in each z_s for which $s \in x^0_{j^{\mathfrak{s}}}$ because $\underline{\eta}$ is increasing in these z_s. It then follows that $E\big(\Delta\pi_{j^{\mathfrak{s}},\,\tau+t^0_{j^{\mathfrak{s}}},\,\Delta t} \mid x^0_{j^{\mathfrak{s}}}\big)$ is increasing in each z_s for $s \in x^0_{j^{\mathfrak{s}}}$ as well. ∎

Proof of Theorem 9

As in the proof of Theorem 8, we use Lemma 2, conditioning on $x^0_{j^{\mathfrak{s}}}$. We have already shown in that proof that $E\Big(\pi_{j^{\mathfrak{s}},t'} \mid x^0_{j^{\mathfrak{s}}},\, \pi_{\rho(j^{\mathfrak{s}}),\,t^0_{j^{\mathfrak{s}}}}\Big)$ is increasing in the quality parameters z_s for $s \in x^0_{j^{\mathfrak{s}}}$. Similarly, $\lim_{t \nearrow t^0_{j^{\mathfrak{s}}}} E\Big(\varpi_{r,t} \mid x^0_{j^{\mathfrak{s}}},\, \pi_{\rho(j^{\mathfrak{s}}),\,t^0_{j^{\mathfrak{s}}}}\Big)$ is increasing in the same parameters because for any submarket $x \in X_{j',\,t^0_{j^{\mathfrak{s}}}}$ that has been discovered by any other firm $j' \neq \rho(j^{\mathfrak{s}})$ that shares an ancestor with $j^{\mathfrak{s}}$, this submarket x shares (at least one of its) attributes with $x^0_{j^{\mathfrak{s}}}$. Both of these variables (with any $t < t^0_{j^{\mathfrak{s}}}$) are, of course, conditionally independent. ∎

An Ancillary Result

Proposition 1 *For any region r, time t and interval $\Delta t > 0$,*

$$\operatorname{cov}\left(\varpi_{r,t},\, \Delta\varpi_{r,t,\Delta t}\right) > 0.$$

Proof Once again, use the general case of Lemma 2, conditioning on $\mathcal{X}_{r,t}$. It is straightforward to verify that current profit $\varpi_{r,t}$ and subsequent growth $\Delta\varpi_{r,t,\Delta t}$ are conditionally independent and that both conditional expectations are increasing in the number of submarkets in the region $\mathcal{N}_{r,t}$ and the quality parameters z_s for all $s \in \bigcup \mathcal{X}_{r,t}$. ∎

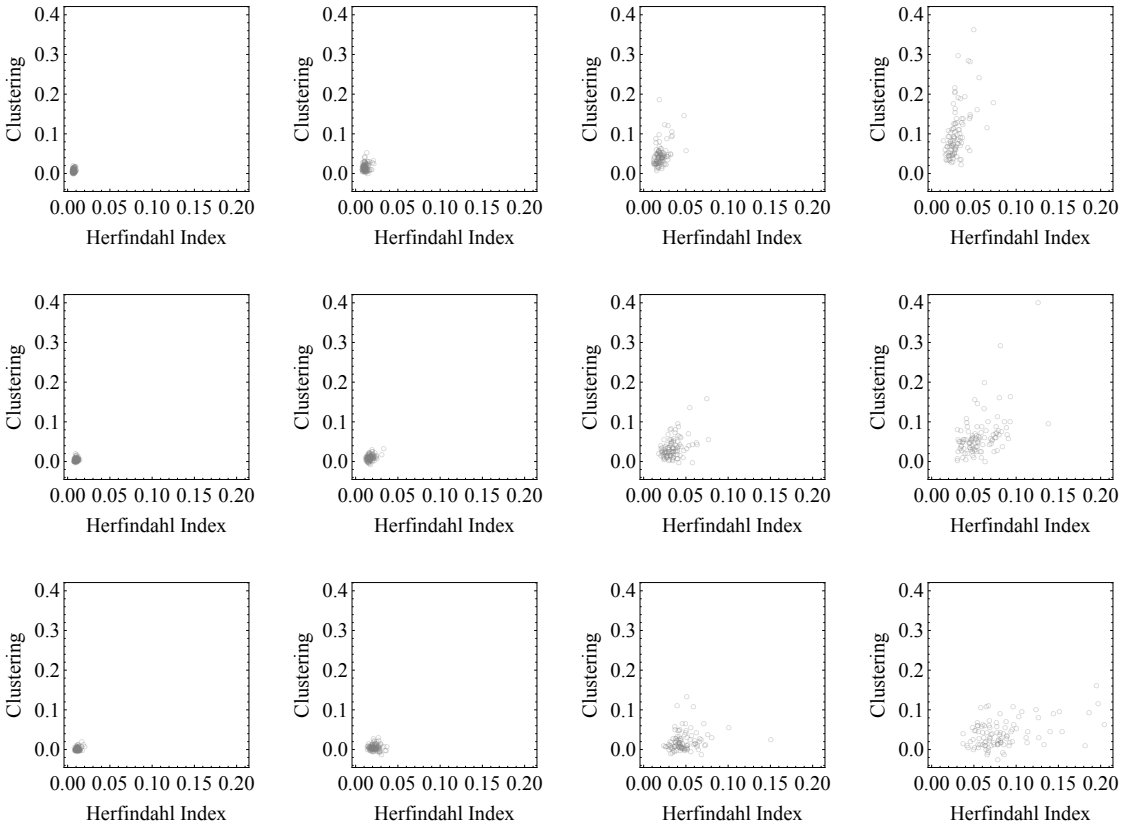

Figure 3: The Ellison-Glaeser index of geographic concentration and the Herfindahl index, recorded upon reaching 1000 submarkets, across all 100 simulated runs of the model. The innovation rate λ varies across the columns, taking on values 0.01, 0.02, 0.05, and 0.1 from left to right. The splitting probability α varies across the rows, taking on values 0.05, 0.1, and 0.2 from bottom to top. We observe positive correlation both within and between panels.

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